Filter optimization for real time digital processing of radiofrequency signals: application to oscillator metrology

A. Hugeat*, J. Bernard†, G. Goavec-Mérou*, P.-Y. Bourgeois*, J.-M Friedt*
*FEMTO-ST, Time & Frequency department, Besançon, France
†FEMTO-ST, Computer science department, Besançon, France
Email: {pyb2,jmfriedt}@femto-st.fr

***Abstract*—Software Defined Radio (SDR) provides stability, flexibility and reconfigurability to radiofrequency signal processing. Applied to oscillator characterization in the context of ultrastable clocks, stringent filtering requirements are defined by spurious signal or noise rejection needs. Since real time radiofrequency processing must be performed in a Field Programmable Array to meet timing constraints, we investigate optimization strategies to design filters meeting rejection characteristics while limiting the hardware resources required and keeping timing constraints within the targeted measurement bandwidths.**

***Index Terms*—Software Defined Radio, Mixed-Integer Linear Programming, Finite Impulse Response filter**

I. DIGITAL SIGNAL PROCESSING OF ULTRASTABLE CLOCK SIGNALS

Analog oscillator phase noise characteristics are classically performed by downconverting the radiofrequency signal using a saturated mixer to bring the radiofrequency signal to baseband, followed by a Fourier analysis of the beat signal to analyze phase fluctuations close to carrier. In a fully digital approach, the radiofrequency signal is digitized and numerically downconverted by multiplying the samples with a local numerically controlled oscillator (Fig. 1) [1].

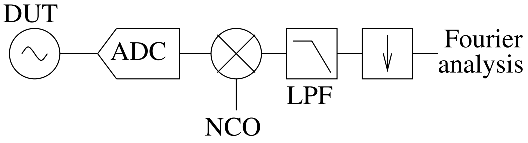

Fig. 1. Fully digital oscillator phase noise characterization: the Device Under Test (DUT) signal is sampled by the radiofrequency grade Analog to Digital Converter (ADC) and downconverted by mixing with a Numerically Controlled Oscillator (NCO). Unwanted signals and noise aliases are rejected by a Low Pass Filter (LPF) implemented as a cascade of Finite Impulse Response (FIR) filters. The signal is then decimated before a Fourier analysis displays the spectral characteristics of the phase fluctuations.

As with the analog mixer, the non-linear behavior of the downconverter introduces noise or spurious signal aliasing as well as the generation of the frequency sum signal in addition to the frequency difference. These unwanted spectral characteristics must be rejected before decimating the data stream for the phase noise spectral characterization. The characteristics introduced between the downconverter and the decimation processing blocks are core characteristics of an oscillator characterization system, and must reject out-of-band signals below the targeted phase noise – typically in the sub -170 dBc/Hz for ultrastable oscillator we aim at characterizing. The filter blocks will use most resources of the Field Programmable Gate Array (FPGA) used to process the radiofrequency datastream: optimizing the performance of the filter while reducing the needed resources is hence tackled in a systematic approach using optimization techniques. Most significantly, we tackle the issue by attempting to cascade multiple Finite Impulse Response (FIR) filters with tunable number of coefficients and tunable number of bits representing the coefficients and the data being processed.

II. FILTER OPTIMIZATION

A basic approach for implementing the FIR filter is to compute the transfer function of a monolithic filter: this single filter defines all coefficients with the same resolution (number of bits) and processes data represented with their own resolution. Meeting the filter shape requires a large number of coefficients, limited by resources of the FPGA since this filter must process data stream at the radiofrequency sampling rate after the mixer.

An optimization problem [2] aims at improving one or many performance criteria within a constrained resource environment. Amongst the tools developed to meet this aim, Mixed-Integer Linear Programming (MILP) provides the framework to provide a formal definition of the stated problem and search for an optimal use of available resources [3], [4].

The degrees of freedom when addressing the problem of replacing the single monolithic FIR with a cascade of optimized filters are the number of coefficients N_i of each filter i, the number of bits c_i representing the coefficients and the number of bits d_i representing the data fed to the filter. Because each FIR in the chain is fed the output of the previous stage, the optimization of the complete processing chain within a constrained resource environment is not trivial. The resource occupation of a FIR filter is considered as $c_i + d_i + \log_2(N_i)$ which is the number of bits needed in a worst case condition to represent the output of the FIR.

The objective function maximizes the noise rejection while keeping resource occupation below a user-defined threshold. The MILP solver is allowed to choose the number of successive filters, within an upper bound. The last problem is to model the noise rejection. Since filter noise rejection capability is not modeled with linear equation, a look-up-table is generated for multiple filter configurations in which the c_i, d_i and N_i parameters are varied: for each one of these conditions, the low-pass filter rejection defined as the mean power between half the Nyquist frequency and the Nyquist frequency is stored as computed by the frequency response of the digital filter (Fig. 2).

Linear program formalism for solving the problem is well documented: an objective function is defined which is linearly

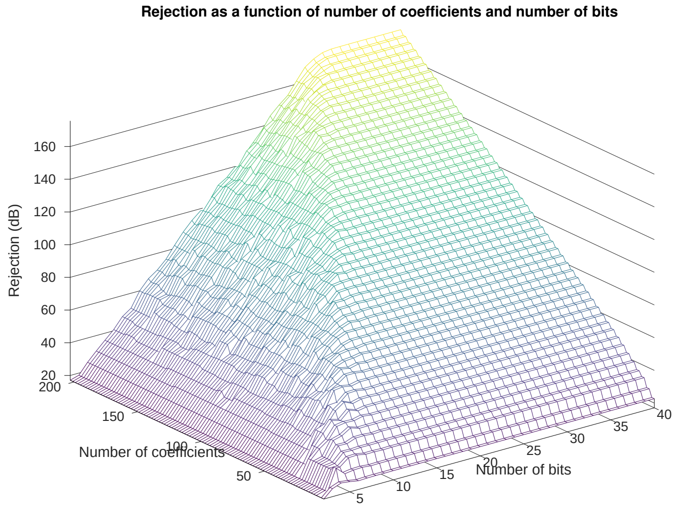

Fig. 2. Rejection as a function of number of coefficients and number of bits

dependent on the parameters to be optimized. Constraints are expressed as linear equation and solved using one of the available solvers, in our case GLPK[5].

The MILP solver provides a solution to the problem by selecting a series of small FIR with increasing number of bits representing data and coefficients as well as an increasing number of coefficients, instead of a single monolithic filter. Fig. 3 exhibits the performance comparison between one solution and a monolithic FIR when selecting a cutoff frequency of half the Nyquist frequency: a series of 5 FIR and a series of 10 FIR with the same space usage are provided as selected by the MILP solver. The FIR cascade provides improved rejection than the monolithic FIR at the expense of a lower cutoff frequency which remains to be tuned or compensated for.

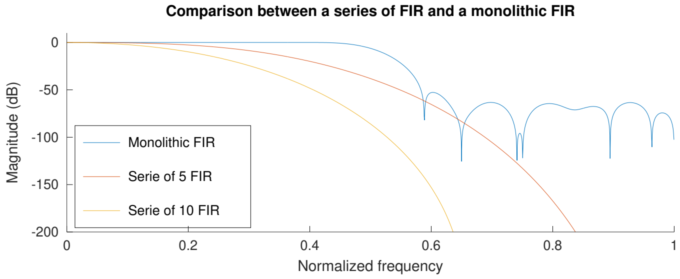

Fig. 3. Comparison of the rejection capability between a series of FIR and a monolithic FIR with a cutoff frequency set at half the Nyquist frequency.

The resource occupation when synthesizing such FIR on a Xilinx FPGA is summarized as Tab. I.

TABLE I

RESOURCE OCCUPATION ON A XILINX ZYNQ-7000 SERIES FPGA WHEN SYNTHESIZING THE FIR CASCADE IDENTIFIED AS OPTIMAL BY THE MILP SOLVER WITHIN A FINITE RESOURCE CRITERION. THE LAST LINE REFERS TO AVAILABLE RESOURCES ON A ZYNQ-7010 AS FOUND ON THE REDPITAYA BOARD.

FIR	BlockRAM	LookUpTables	DSP
1 (monolithic)	1	4064	40
5	5	12332	0
10	10	12717	0
Zynq 7010	60	17600	80

III. FILTER COEFFICIENT SELECTION

The coefficients of a single monolithic filter are computed as the impulse response of the filter transfer function, and practically approximated by a multitude of methods including least square optimization (Matlab's `firls` function), Hamming or Kaiser windowing (Matlab's `fir1` function). Cascading filters opens a new optimization opportunity by selecting various coefficient sets depending on the number of coefficients. Fig. 4 illustrates that for a number of coefficients ranging from 8 to 47, `fir1` provides a better rejection than `firls`: since the linear solver increases the number of coefficients along the processing chain, the type of selected filter also changes depending on the number of coefficients and evolves along the processing chain.

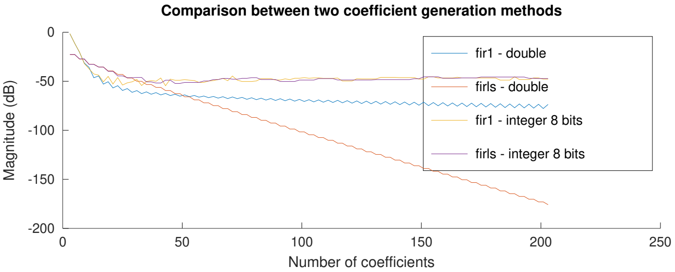

Fig. 4. Evolution of the rejection capability of least-square optimized filters and Hamming FIR filters as a function of the number of coefficients, for floating point numbers and 8-bit encoded integers.

IV. CONCLUSION

We address the optimization problem of designing a low-pass filter chain in a Field Programmable Gate Array for improved noise rejection within constrained resource occupation, as needed for real time processing of radiofrequency signal when characterizing spectral phase noise characteristics of stable oscillators. The flexibility of the digital approach makes the result best suited for closing the loop and using the measurement output in a feedback loop for controlling clocks, e.g. in a quartz-stabilized high performance clock whose long term behavior is controlled by non-piezoelectric resonator (sapphire resonator, microwave or optical atomic transition).

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REFERENCES

[1] J. A. Sherman and R. Jördens, "Oscillator metrology with software defined radio," *Review of Scientific Instruments*, vol. 87, no. 5, p. 054711, 2016.
[2] J. Y. Leung, *Handbook of scheduling: algorithms, models, and performance analysis*. CRC Press, 2004.
[3] Y. J. Yu and Y. C. Lim, "Design of linear phase FIR filters in subexpression space using mixed integer linear programming," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 54, no. 10, pp. 2330–2338, 2007.
[4] D. Kodek, "Design of optimal finite wordlength FIR digital filters using integer programming techniques," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 28, no. 3, pp. 304–308, 1980.
[5] "https://www.gnu.org/software/glpk/," availble online, accessed Jan. 2018.